

# ACTIVITIES THAT ILLUMINATE AND VISIBILIZE FUNCTIONS AS OBJECTS

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*Purpose of the workshop is to afford teachers from the Wits Maths Connect professional development a space to share their PD experiences with colleagues. In the proposed workshop teachers will present to Grades (9 - 12) mathematics teachers some novel ways of teaching Functions. These are ways that aim to help learners perceive functions as constituting a relationship. The workshop will have three foci. First foci will engage participants in an activity that provides teachers with ways of seeing and explaining the horizontal shift in Functions. Second foci will engage participants with ways of helping learners move from perceiving functions as process to seeing them also as objects. The third foci will engage participants in an activity that illuminates and visibilizes the relationship between functions, domain and range. The workshop will cater for 30 teachers who will work in groups of 5 on selected activities. The duration of the workshop is 2-hours. At the beginning of the workshop we will use 10 minutes to introduce the workshop. Workshop activities will be allocated 1hr 20 minutes and the remaining 30 minutes will be used to reflect on the activities and for general discussions about the workshop.*

## **MOTIVATION FOR THE WORKSHOP**

This workshop is important for participants since it shares experiences/ideas and novel ways of explaining the Horizontal Shift in Functions. Functions are one of the topics in the mathematics curriculum which presents most challenges to learners. There is sufficient evidence that suggests that majority of our learners perform below national expectations in this area of mathematics (Adler, 2010; Simkins, 2013; Spaul, 2013). The 2012 report on the National Senior Certificate Examination (DoE, 2012) provides information that emphasizes the magnitude of the problem of low performance. We have selected to focus our workshop on Functions on account of its role as a gateway to University mathematics.

The essence of Functions in secondary school mathematics is a passage learners need to navigate in order to access further and harder university mathematics (Tall, 2010a). As such, there is greater need, through congresses such as AMESA, for teachers to share novel ways of teaching to minimize obstacles learners encounter along the passage to higher mathematics. We believe the workshop brings an alternative way of teaching Functions in ways that might minimize these obstacles, and that the workshop will provide teachers with opportunities to improve their teaching quality and increase learning opportunities for learners, particularly in the domain of functions. One of the obstacles we hope to address is the one where learners view graphs of Functions as pictures rather than relationships (Adler, 2010) .

In the Wits Maths Connect (WMC) project, we have noticed learners failing to obtain a solution from two intersecting graphs, preferring to solve equations of the graphs simultaneously which in our opinion was an unnecessary waste of time since we thought it could be much easier and quicker to obtain the solution from the intersection of two graphs. Now what was amazing was the fact that most learners, who had chosen the algebraic manipulation route, could not obtain the correct solution. We believe that learners could not obtain solutions from the graph because they perceived it as a picture. We associate learners' perception of graphs of Functions as pictures with teaching that always move from algebraic equation, to producing a table of values, to plotting and joining of points to produce a graph and where the graph is the product. We think this approach obscures the essence of Functions, (i.e. Functions as relationships between variables) from learners. In the workshop we propose to demonstrate how Functions may be taught in ways that might enable learners to see graphs of Functions as representing relationships that can be studied and to see them as having properties that can be analyzed to obtain solutions (Tall, 1997a). But most importantly we intend to demonstrate how all the four ways in which Functions may be represented are independent of each other and can be used to complement each other.

## DESCRIPTION OF WORKSHOP CONTENT

The content of the workshop will be broken down as follows:

### **Introduction:** (10 minutes)

The workshop will begin with an introduction of presenters and this will be followed by explanations about the rationale (for selecting Functions; specific content on Functions; our approach; and target group), purpose, and the design of workshop activities.

### **Workshop Presentation:** (1hr 20 min)

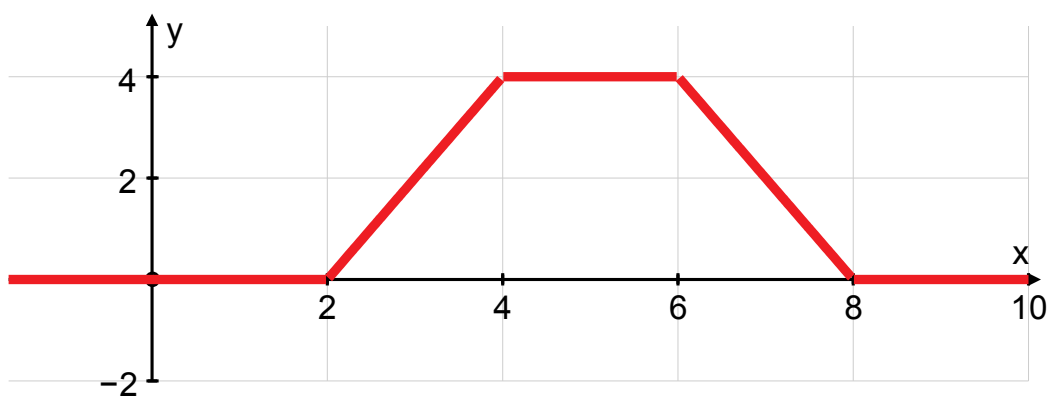
**Target Audience:** Our main target is teachers offering mathematics at Grades (9 -12)

**Duration:** The workshop is designed to fit within a 2-hour Slot

**Maximum no. of participants:** Thirty (30)

**Activity 1:** Investigating the horizontal shift using a piecewise function (30 minutes)

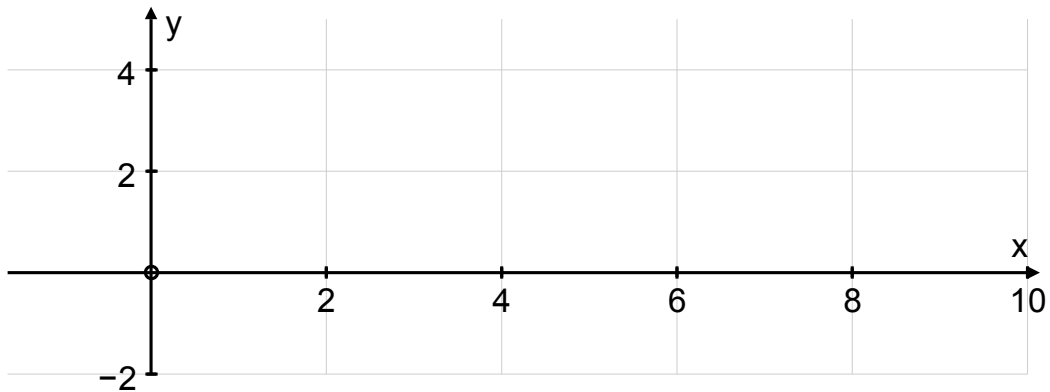
In this activity participants will be given non-standard or piecewise functions and translation rules to observe and analyze the horizontal shift on the given graphs. The benefit of this activity is that it provides the explanatory framework which teachers can use to explain how and why the graph of  $g(x)$  might when transformed to  $g(x-1)$  and  $g(2x)$ . **Figures 1, 2, 3** and **Table 1** are examples of resources to be used in Activity 1.



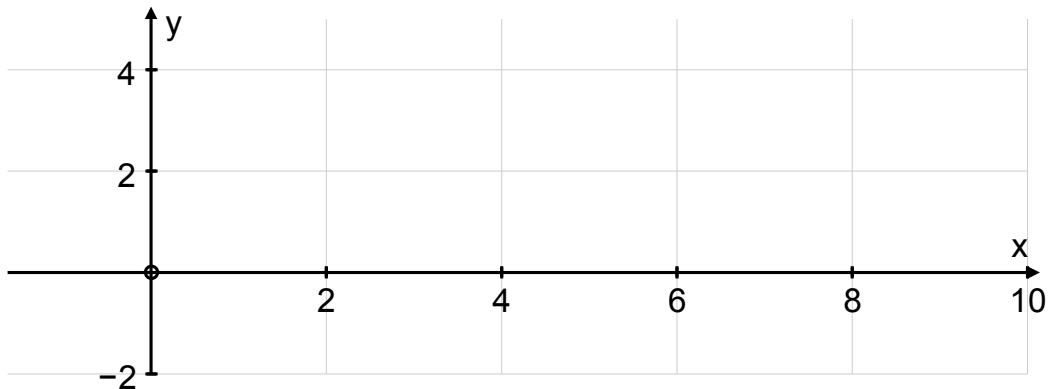
**Figure 1: Graphical representation of the piecewise Function**

$x$	0	1	2	3	4	5	6	7	8	9	10
$g(x)$											
$g(x-1)$											
$g(2x)$											

**Table 1: Numerical representation of the piecewise Function**



**Figure 2: Cartesian plane for the graph of  $g(x-1)$**



**Figure 3: Cartesian plane for the graph of  $g(2x)$**

**Activity 2:** Moving from functions as process to functions as objects (30 minutes)

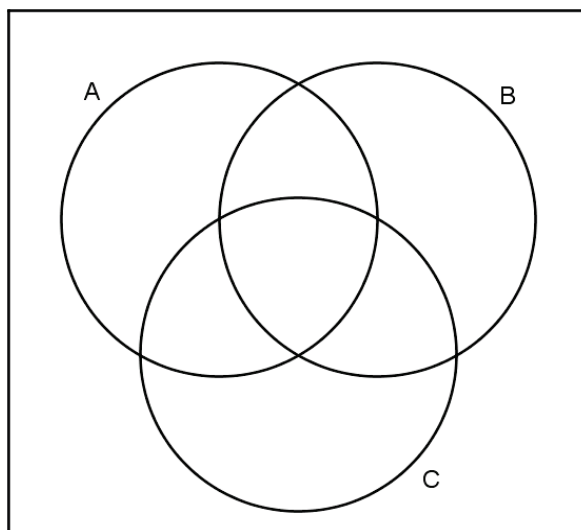
In this activity participants, working in groups of 5, will be issued with worksheets containing four conditions. Participants will then be required to find 8 functions that satisfy the given conditions. The activity requires participants to know and work with properties of functions simultaneously; hence when used in the class it will facilitate learners' global view of functions. Thus, a move from functions as process to functions as objects. **Figure 4** is an example of a resource for Activity 2.

Fill the spaces in the three intersecting circles with functions that satisfy these three conditions

A: The range includes the value  $y = -1$

B: The graph is of the form  $y = f(x-3) + a$

C: The graph passes through the fourth quadrant



**Figure 4: Intersecting circles**

**Activity 3: Matching functions with graphs, domain and range (30 minutes)**

Working in groups of 5, participants will be provided with 36 cards containing function, graph, domain and range. Each group will sort the cards into piles of four such that each pile contains a function and its matching graph, domain and range.

**SUMMARY AND CONCLUSION**

**Activity: Participants' reflections on activities and discussions: (20 minutes)**

In this part of the workshop, participants will reflect on the 3 activities, discuss and share experiences drawn from engaging with workshop activities.

## ACKNOWLEDGEMENTS

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